CAN PROBLEM SOLVING AFFECT THE UNDERSTANDING OF RATIONAL NUMBERS IN THE MIDDLE SCHOOL SETTING?

A Thesis

by

KRYSTAL BAKER MEREDITH

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

May 2009

Major Subject: Curriculum and Instruction

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Approved by:

Co-Chairs of Committee,	Robert M. Capraro
	Mary Margaret Capraro
Committee Members,	Bruce Thompson
Head of Department,	Dennie Smith

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ABSTRACT

Can Problem Solving Affect the Understanding of

Rational Numbers in the Middle School Setting? (May 2009)

Krystal Baker Meredith, B.S., Texas A&M University

Co-Chairs of Advisory Committee: Dr. Robert M. Capraro
Dr. Mary Margaret Capraro

In this study, problem solving provided deeper meaning and understanding through the implementation of a structured problem solving strategy with the teaching of rational numbers. This action-research study was designed as a quasi-experimental model with a control closely matched to an experimental group using similar demographics and number of economically disadvantaged students. In comparison to the control group, the experimental group received their instruction in rational numbers with the addition of a structured problem solving strategy, and a pre/posttest on problem solving with proportionality between similar geometric figures, converting fractions to percents, proportionality with a given ratio, expression of a ratio, and appropriate application of ratios. The study indicates that a structured problem solving strategy can improve the mathematical accuracy, approach and the explanation of rational numbers that are focused on rates, ratio, proportion, and percents. Results showed a statistically significant difference in the performance of these two groups. Effect sizes and 95% confidence intervals (CIs) were reported to support the findings.

When examining subgroups, the study showed the structured problem solving stratey not only improved students' ability to understand and use rational numbers but also improved students' problem solving skills and their attitude towards problem solving. The experimental group showed the most improvement in the approach to solving problems with rational numbers indicating deeper understanding of rates, ratios, proportions and percents.

DEDICATION

To my husband, Mr. Wonderful

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CHAPTER I

INTRODUCTION

While observing students in my middle school mathematics classes, I noticed students were struggling with various areas of content. I realized that the students in my classes had not been taught how to use problem solving strategies when dealing with problems, more specifically those involving rational numbers (Lesh, Post, & Behr, 1987). According to the National Council of Teachers of Mathematics (NCTM) problem solving allows students to “experience the power and utility of mathematics” (NCTM, 2000, p. 256). Based on this view of problem solving, the ultimate goal of this study was to improve student scores and understanding of rational numbers by implementing systematic, thoughtful problem solving into middle school classrooms.

Historically, problem solving has been viewed as a discrete set of skills with many different processes (Schoenfeld & Herrmann, 1982) and more recently problem solving has been viewed as a gatekeeper to taking more advanced high school mathematics (Baroudi, 2006; Gagnon & Maccini, 2001). Problem solving provides an atmosphere where students who have difficulties reaching a solution based on an algorithm can explore alternate solution methods and incorporate manipulatives, graphic organizers, and visualizations. This leads students to a better understanding of the

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mathematics and the construction of how mathematics is related to personal experiences because the student is forced to focus on the ideas while engaging in the activity (Van de Walle, 2007). Students begin to feel success in mathematics due to the nature of problem solving being about the process and explanation not necessarily the solution being "right" (Gagnon & Maccini, 2001; Van de Walle)

Research has demonstrated that there was a relationship between the different perspectives of rational numbers when students used them during problem solving (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Moseley, 2005). Students were able to group rational numbers into multiple perspectives, part-whole, quotient, ratio, measure, and operator, after problem solving exercises showing organization of their knowledge about the perspectives of rational numbers as a concept that was interconnected rather than separate ideas (Ben-Chaim, et al., 1998; Carraher, 1993; Moseley, 2005). This can then be taken and used to teach proportional reasoning with understanding at multiple representations and levels (Behr, Lesh, Post, & Silver, 1983).

Statement of Problem

The NCTM calls educators to deepen middle school students' understanding of rational numbers in the *Principle and Standards for School Mathematics* through "extensive experience with ratios, rates, and percents" (2000, p. 215). The mastery of rational numbers begins with a relationship of part to whole in multiple representations such as decimals, ratios, rates, proportions, and percents. In my observations and reflections as a middle school mathematics teacher, students struggle with identifying part and whole relationships within rational numbers. Students stumble when translating rational numbers and when attempting to illustrate them with multiple representations. Brown and Quinn (2006) claimed that students struggled with understanding rational numbers as a result of "no connection for understanding" because the student has only been taught the algorithm" (p. 28).

Although students struggle with problem solving in the middle grades, it can offer a connection to the learning and understanding of rational numbers (NCTM, 2000). Through problem solving, students are provided the opportunity to work with algorithms and rational numbers in settings that have meaning and application. Problem solving does not limit students to one algorithm thus providing students with a comfortable atmosphere where they can explore rational numbers and have actual experiences with their multiple uses.

Rationale

Rational numbers are presented in various forms, such as, fractions, rates, ratios, percents, proportions, and scale. All of these forms of rational numbers are interrelated but begin on the basis of a relationship of part to whole between two numbers. This relationship is often difficult for students to interpret and understand because they have not had the opportunity to apply the relationship or they do not understand the uses of the relationship. Many educators are at fault, because they simply draw on the algorithm for solving or finding the rational numbers. This creates a lack of understanding for the student by limiting their connections to prior knowledge as well as connections to experiences.

In an effort to provide connections and to improve student understanding of rational numbers I used problem solving to bring problems to life. Students were presented problems in a realistic setting, allowing them to pull from their independent and collaborative experiences (Stillman & Galbriath, 1998). They were encouraged to explain and share their answers to encourage deeper connection and understanding of rational numbers. The explanation often required students to answer the question "Why?" which encouraged students to see the relationship between the numbers and reinforced their understanding of rational numbers.

Significance of Study

Problem solving assisted understanding by providing the platform for students to openly explore their perceptions and learn about the applications of rational numbers. Problem solving provides insightful learning by allowing student's approach to influence their independent construction of meaning and how various models of rational numbers are interrelated.

Students' understanding of rational numbers and the need to problem solve becomes more necessary as they progress in their mathematical education. Rational numbers are needed to be successful in the community outside the mathematics classroom. Rational numbers lead students to their future in mathematics and assists them in transitioning from arithmetic uses of whole numbers to the deepened understanding of mathematics (Lamon, 1993).

Research Questions

The following questions framed this study:

1. Does the implementation of a structured problem solving strategy affect the understanding of rational numbers with middle grades students?

2. Does the implementation of a structured problem solving strategy affect students' performance in problem solving?

3. Can the implementation of a structured problem solving strategy influence students' attitudes about problem solving?

Definition of Terms

Rational numbers: numbers that are based on a relationship or a comparison between numbers, which means that rational numbers include representations of fractions, decimals, indicated division, percents, ratios, rates, and proportions (Behr, et al., 1983; Van de Walle, 2007).

Problem Solving: interaction in a task or with a problem where the "solution method is not known in the advance" (NCTM, 2000, p. 52).

Problem posing: fostering the idea that students can generate problems based on independent situations or that students can reformulate a previously encountered problem (NCTM, 2000; Stoyanova, 2003).

Middle school student: the young adolescent student in grades 6 through 8 (NCTM, 2000).

Before students can progress from arithmetic operations in elementary mathematics to algebraic reasoning in middle school mathematics classrooms, students must first understand that there are many different kinds of numbers. Numbers can be seen and viewed from a multitude of facets. The specific facet that this study focuses on is rational numbers and how ratios and proportional reasoning can be influenced by problem solving.

CHAPTER II

REVIEW OF LITERATURE

Rational Numbers

NCTM mandates that students should have “extensive experience with ratios, rates and percents, which helps form a solid foundation for their understanding of, and facility with, proportionality” (NCTM, 2000, p. 215). The Texas Education Agency (TEA, 2006) supports the teaching of rate, ratios and proportions at the middle school level in their publication of the *Texas Essential Knowledge and Skills* (TEKS). The TEKS affirm that sixth grade students are expected to master rates and ratios in a proportional relationship, identify ratios as fractions, percents and decimals and use proportional reasoning to predict a ratio or percent (TEA). This objective is structured to support the NCTM number and operations strand that includes rate, ratios and proportions. NCTM pushes the TEKS initiative further and advocates using rates, ratios and proportions to facilitate learning in other content strands in the middle school curriculum. Rates, ratios, and proportions can be used to implement instruction in measurements, comparison of similar figures, scale factors, rate of change, fractions, and slope in linear functions (Brown & Quinn, 2006; Cramer & Post, 1993b; NCTM, 2000).

Rational number concepts and proportional reasoning are difficult for students to grasp (Brown & Quinn, 2006; Cramer & Post, 1993b; Cramer, Post, & delMas, 2002; Kilpatrick, Swafford, & Findell, 2001; Van de Walle, 2007). Before we address the difficulty that students experience with rational numbers we must first give them a definition. Rational numbers are based on a relationship or a comparison between numbers, which means that rational numbers include representations of fractions, decimals, percents, ratios, rates, and proportions (Behr et al., 1983; Brown & Quinn, 2006; Kilpatrick et al., 2001; NCTM, 2000; Van de Walle, 2007). When using a fraction as a rational number, the student is comparing part-to-whole and examining the relationship between two numbers where the whole can be divided into any number of parts (Cramer et al., 2002; NCTM, 2000). When examining decimals within rational numbers, students are looking at the part in relation to the single unit whole in base 10 where the whole is only divided into powers of ten (Kilpatrick et al., 2001; NCTM, 2000; Van de Walle, 2007). Percents evaluate the relationship between the part and whole in relationship to one hundred or percent (NCTM, 2000; Van de Walle, 2007). Ratios allow students to examine the relationship between numbers without being exclusive to the part-to-whole relationship. When working with ratios, students can examine the relationship of part-to-part or part-to whole. Rate is a specialized ratio that allows students to compare two different measures. Rather than a part-to-whole or part-to-part relationship, students can begin to examine relationships of unrelated measurements such as miles-to-gallons or feet-per-second. This leads students into proportional relationships where the multiplicative operation can be used to infer what would happen if the same conditions were presented with a specific ratio on a larger or smaller scale. The relationship then was not only between each part of the ratio but also a relationship of equivalence between two ratios (Behr et al., 1983; Cramer & Post, 1993a; Ben-Chaim et al., 1998; NCTM, 2000; Van de Walle, 2007). Although fractions and decimals are important and lend towards basic understanding of rational numbers, this study focused on the use of rates, ratios, proportions and percents in a sixth grade middle school classroom.

Middle grades students tend to have difficulty facilitating rational numbers. These students struggle seeing all the aspects of rational numbers, how the various aspects overlap and how each representation is a reflection of the relationship between the numbers (Ashlock, 2006; Flores & Kaylor, 2007; Van de Walle, 2007). Students tend to separate and compartmentalize ratios, percents, fractions and proportions into separate ideas (Ashlock, 2006; Heller, Ahlgren, Post, Behr, & Lesh, 1989; NCTM, 2000; Van de Walle, 2007). The disjointed understanding of rational numbers began with textbooks that lack the components that are necessary for effective learning (Cramer & Post, 1993a; Flores & Kaylor, 2007). Many times students were simply taught the algorithmic method of cross-products for solving a proportion or for comparing ratios, leaving the students with only a series of steps to follow (Behr et al., 1983; Brown & Quinn, 2006; Cramer & Post, 1993a; Van de Walle, 2007). As a result, students failed to understand the meaning behind the numbers, computation, the multiplicative relationship or the equivalence relationship (Bay-Williams & Martinie, 2003; Brown & Quinn, 2006; Cramer & Post, 1993a; Lesh, Post, & Behr, 1988). As rational numbers and proportional reasoning become necessary in future learning, students then fail to make connections between the relationships and struggle with other areas of curriculum where these concepts overlap or provide support (Ashlock, 2006; Cramer & Post, 1993b).

When teaching ratios and proportional reasoning to middle school students, educators should provide, “the opportunity to compare two related quantities, reason about the relationship between them, and develop their own methods of comparison" (Bay-Williams & Martinie, 2003, p. 286). Students should begin their experience with ratios by utilizing manipulatives or visualizations that are familiar (Ashlock, 2006; Cramer & Post, 1993a; Huetinck & Munshin, 2004; Van de Walle, 2007). Concrete examples provide an opportunity for students to focus on the relationships between the quantities as well as the equivalence relationship between the ratios (Ashlock, 2006; Huetinck & Munshin, 2004). Concrete manipulatives can still provide support for further learning when the computational algorithm is taught by relating the computation and algorithms to the manipulatives and relationships. According to Cramer and Post, this means that educators "will have to go beyond the content of textbooks to offer meaningful instruction for this important domain" (1993a, p. 407).

Students should be exposed to proportional reasoning with problems that are based on missing-value, numerical comparison, and qualitative prediction and comparison. These various types of problems do not steer students directly into the algorithm of cross-products. Missing-value problems require students to use the relationship in a ratio to infer the missing value in an equivalent ratio. Numerical comparison problems ask students to evaluate or observe a set of ratios and determine which ratio represents a larger or smaller quantity. Qualitative prediction and comparison problems question a student's understanding of proportions to infer limitations for the problem and reasonableness (Ben-Chaim et al., 1998; Cramer & Post, 1993a,1993b; Van de Walle, 2007).

Problem Solving

The NCTM defined problem solving as, “engaging in a task for which the solution method is not known in advance” (2000, p. 52). With problem solving, students are required to utilize their individual knowledge and experiences to develop a solution and mathematical understanding (NCTM).

Polya (1957) approached problem solving by acknowledging that students need to be taught a strategy to assist with the construction of their mathematical knowledge with guidance from the teacher. Polya began his problem solving strategy by stating that all students should *understand the problem*. This first step of solving a problem was to understand all of the parts of the given problem and the factors that may have influenced the solution. Students were to rely on prior knowledge and experience to ensure that they understand what the problem is asking. Students should have also incorporated any outside information that could affect the problem, the purpose of the given information and attempt to simplify the problem if possible (Polya, 1957). Once the problem solver understood the problem they moved to *devise a plan* according to Polya. The purpose of this step was to develop a plan of action for the solution. The problem solver should take all the information in the understanding of the problem combined with personal experience and devise a plan to efficiently reach the solution. The next part of the problem solving strategy provided an opportunity for the problem solver to work the problem. Polya’s problem solving strategy called this step *carryout the plan*. This referred to the actual working out of the problem. The final step of Polya’s problem solving strategy is *looking back*. This final step was where the problem solver reviewed their problem for reasonableness, additional solution methods, and checked for errors.

Like Polya (1957), Schoenfeld's (1980) problem solving strategy began with the first step of understanding what the problem asked with a step called *analysis*. Students were given directives to consider if the problem can be simplified or broken into smaller simpler problems, as well as, what other information can be applied to the problem based on prior learning (Schoenfeld,1980). The next step in Schoenfeld's problem solving strategy was called *design*. Like Polya, the purpose of *design* was to create a plan of evaluation for the problem. Schoenfeld added an additional step during and just after the design phase of his strategy called *exploration*. Schoenfeld's exploration step called on the problem solver to consider and attempt problems that were "essentially equivalent", "slightly modified", or "broadly modified" (p. 801) to the initial problem. This step was where students were encouraged to apply the similarities from these problems to their plan for solving the initial problem. Students traveled back and forth between the *design step and exploration* several times as they built and modified their plan of evaluation. Schoenfeld's next step in his problem solving strategy was called *implementation*. This step was the working of the initial problem that was given. The final step of Schoenfeld's problem solving strategy was *verification*. Like Polya, students were encouraged to explore alternative solution methods, check their solution for reasonableness and verify that the solution does not have computational errors (Polya1957; Schoenfeld, 1980).

Although, both Polya (1957) and Schoenfeld (1980), provided a step to wrap up the problem and explore additional solution methods, possible applications of the problem, and checking for accuracy, neither plan provided a step where the problem solver or student was required to explain their solution. The student did not consciously acknowledge their metacognition during the problem solving process and acknowledge their reasons for their solution or reasons for rejecting a process for the solution.

Kulm and Bussman (1980) provided a structured problem solving strategy that incorporates eight phases. These phases did not have to be followed sequentially and some phases may be worked as a part or integrated into another phase. The phases began with understanding the problem similar to Polya (1957), then moving to analysis of the problem and exploring ideas for the solution. After analyzing the problem, the problem solver began to anticipate problem elements and create similar problems in a simpler case. The next phase discussed anticipating new patterns in the problem and thinking beyond the specific situation. With the fifth phase, concepts for the problem were refined to help bring clarity to the problem leading to the next phase where the problem solver began to formulate a solution. The concepts formulated in the previous phases are then related back to the original problem to work out and establish a solution. The final phase was where students checked and reflected on their problem solving approach and assess new ideas and concepts. Kulm and Bussman stated that “any phase can be repeated and that a particular phase might be skipped, then eventually carried out later or incorporated in another phase” (1980, p.186).

Kulm and Bussman's (1980) phases were similar to Schoenfeld's (1980) problem solving strategy by allowing the students to cycle through several phases or steps, having a free flowing motion back to other parts of the strategy. However, their phases did not advocate the problem solver explaining and rationalizing why they were pursuing a specific route or plan. The problem solver reflected on their journey through the phases and had the opportunity to assess new ideas, but they were not required to explain why it was applicable to the solution

Charles and Lester (1984) designed and implemented a structured problem solving strategy that was similar to Polya (1957) and Schoenfeld (1980). The structured problem solving strategy was modeled after Polya's strategy specifically with the first step being *understand the problem*, This section requested that students reread the problem, rephrase the problem and to identify what the student was trying to find or identify. Charles and Lester (1984) did not provide a planning step or encourage students to develop a plan for their solution. The next step in their problem solving strategy was *solving the problem* where students are provided with a series of heuristic approaches for solving problems to select or choose. The last step in the problem solving strategy of Charles and Lester was *answering the problem*. In this final step students were encouraged to check their work, check the solution for reasonableness, and write the solution in complete sentences. Charles and Lester (1984) provided a lesson plan and teaching guide for teachers in the use of this problem solving strategy that encouraged discussion and small group work, but they did not require any written explanation of the steps for the problem or an explanation of the solution.

Stillman and Galbraith's (1998) focus on problem solving was that the attempt at the process was as important as the attempt of the solution. These researchers supported this when using the *information processing approach* to problem solving. This processing approach was a cyclical one whereby any part can be cycled through multiple times or not at all from other steps. This process for problem solving began with *information gathering* that was similar to Polya's *understand the problem* (1957). The student selected the necessary information from the problem to reduce and interpret the problem. The next component for solving a problem was the *information representation* whereby a student called on previous experiences to organize the information from the problem. This step was followed by *search and information processing* similar to Polya's *devise a plan* due to the idea that the problem solver was processing the information and beginning to work out a plan for the solution (1957). The final step of the information processing approach to problem solving called for *information validation* where the problem solvers justified their steps and their solutions (Stillman & Galbraith, 1998).

Cifarelli (1998) did not focus on a problem solving strategy with his students but on their explanations and how they changed their representations and internal structures over the process of solving problems. Cifarelli did not tell his participants to problem solve using a specific method or strategy, rather he had the participants talk out their solutions to allow the researcher to hear their conceptual knowledge, as well as, understand the reasoning and explanation for using any specific method for a solution

(Cifarelli & Cai, 2005a, 2005b). It was intriguing how the students' views and internal connectedness of knowledge changed over the course of the study giving validity to the idea that explaining and rationalizing your solution strategies can help establish and enhance your problem solving abilities. Cifarelli and Cai (2005a) showed that problem solving was a cyclical routine where students posed problems related to the initial problem to organize and structure the information then solve the problem based on the goal that was initiated in the problem posing process (Cifarelli & Cai, 2005b). Once the goal was reached, the problem solver began the process again by posing a new problem and seeking a solution for that goal, thus continually repeating the process.

Cifarelli (1998), similar to Polya (1957) and Schoenfeld (1980), felt that the problem solving process was cyclical and that it could change and morph as a student reaches a solution. However, Cifarelli did not feel a prescriptive process was necessary for students in their problem solving process. Allowing students the time and opportunity to work with open-ended problems where they used their current knowledge and skills to pose problems and reorganize solutions created deeper mathematical meaning and stronger problem solving skills (Cifarelli & Cai, 2005a).

Ferruci, Yeap, and Carter (2003) approached problem solving from the point of implementation through modeling. These researchers stated, "traditional arithmetic word problems induces in pupils a mindless, superficial, routine-based way to identify the correct arithmetic operation needed to solve a word problem" (p. 470). However, this thoughtless strategy was not helping students become successful problem solvers (Jitendra, Griffin, Haria, Leh, Adams, & Kaduvettoor, 2007; Lesh, Post, & Behr, 1987).

Many students did not understand the purpose of going through this routine or the expected outcome that should be learned (Lubienski, 2000). Students were simply applying a method without thought to obtaining an answer (Lesh et al., 1987). Modeling problem solving assisted students in making a connection to the method of solving problems and later applying the strategy in other areas of mathematics (Adibnia & Putt, 1998; Ferrucci, Yeap, & Carter, 2003; Stoyanova, 2003).

Problem Solving as Problem Posing

Some researchers believe the idea that mathematical problem solving skills can be strengthened by problem posing. Stoyanova (2003) identified problem posing as the, “creation of a new problem from a situation or experience, or reformulation of given problems” (p. 33). Linking a student’s “personal interests with their education” (p. 33) was the primary goal of problem solving and problem posing (Bottge, Rueda, & Skivington, 2006). The link between the classroom and the outside world allowed the “students’ experiences in mathematics classrooms help them to become competent users of mathematics by being able to pose, analyze and solve real world problems” (Stoyanova, 2003, p. 33). Problem solving became realistic and the students focused on investigation and explanation of problems rather than solutions (Bottge et al., 2006).

Problem posing can produce better problem solvers by strengthening their analytic abilities (Flores, Turner, & Bachman, 2005; Stoyanova, 2003). The idea that the process for the solution of a problem held the same importance as the problem’s solution was reflected in Stillman and Galbraith’s (1998) research. These researchers claimed that implementing problem solving through posing problems in a “real world” setting strengthened students' ability to relate and solve problems. Problem solving required students to apply "real world knowledge in the solution process" (Stillman & Galbraith, 1998, p. 158) concluding that posing real world problems was part of the journey with problem solving. These researchers based the success of problem solving on the students' experiences as they "make use of the available cognitive and metacognitive resources" to successfully come to a solution of a problem (Stillman & Galbriath, 1998, p. 186).

Students struggled when problem solving was under the constraints of the classroom curricula (Lesh et al., 1987). They were not allowed to use all their resources or creativity to come to a solution. As a result, students tended to be frustrated with regimented mathematical problem solving tasks (Bottge et al., 2006; Lubienski, 2000). Jurdak (2006) suggested that adding situated problem solving or problem solving based on real life experiences helped change this attitude when he stated, "situated problem solving may have an attitudinal effect regarding real life problem solving" (p. 297). Some differences may have existed between the classroom solution process and the realistic solutions but students were allowed to see how mathematical problem solving could be meaningful outside the classroom (Jurdak, 2006). Students had a stronger desire for problem solving in the classroom when they could see and use the mathematics in situations that they had experienced or will experience (Ben-Chaim et al., 1998; Bottge et al., 2006; Jurdak, 2006; Stillman & Galbraith, 1998).

Cifarelli and Cai approached problem posing as the task "that involves the solver's interpretations and how they give meaning to the tasks" (2005b, p. 1). As students continually reformulated and restructured their ideas and solutions for solving a problem, they posed new problems and related the solutions and structure to the cyclical problem solving process. This definition of problem posing was the idea that students related their knowledge and experiences to the evolving problem solving process and redeveloped the problem continuously by posing problems then solving the problem (Cifarelli & Cai, 2005a, 2005b).

Flores et al. (2005) viewed problem posing similarly to Cifarelli and Cai (2005a, 2005b) Their research supported the idea that problem posing could provide insight and conceptual understanding for students as they reconstructed their current knowledge. Problem posing provided an arena where the rules did not exist thus allowing students and teachers to approach problems as a way to define and make sense of mathematics (Flores et al., 2005).

Problem Solving in Middle School

Problem solving promoted students' exploration of independent solutions to a problem and finding what the solution means to them (Ben-Chaim et al., 1998). When students explored problems using their own methods and there were more open mathematical solutions, students built creative solutions (Herman, 2007; Rittle-Johnson & Star, 2007). The Connected Math Project (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006) supported these findings by basing its curriculum wholly on students learning mathematics by solving problems in interesting settings and providing students the opportunity to observe patterns, relationships, and develop critical thinking skills (Ben-Chaim et al.). By allowing evaluation of multiple methods and solution processes through problem solving, students used these experiences to practice “symbolic notation” (Baroudi, 2006; Rittle-Johnson & Star, 2007).

Problem solving allowed students to use models and representations with their solutions that helped them relate to their real experiences (Baroudi, 2006; Bottge et al., 2006; Uesaka, Manalo, & Ichikawa, 2007). This relationship helped students correct the arithmetical misconceptions they had previously made by allowing them to see the correct arithmetic meaning in the context of problem posing (Baroudi, 2006; Brown & Quinn, 2006; Gagnon & Maccini, 2001).

Bottge et al. (2006) found that problem solving based on “enhanced anchored instruction” provided an opportunity for students with emotional and behavioral disabilities to gain basic arithmetic skills and confidence in their ability to work with rational numbers and measurement. In this study students worked with video-based instruction for building a skateboard ramp and a hovercraft. The problems initially caught the students’ focus because they were interested in the construction and hands-on activities connected to the problems. Students drew scale plans, budgeted materials, and worked with others on these constructions. This problem-solving approach demonstrated that problem solving not only can enhance some students’ abilities to connect with mathematics but all students’ abilities to connect with mathematics (Bottge et al., 2006).

Gagnon and Maccini (2001) made similar claims for problem solving and students with disabilities in classrooms. These students often struggled with arithmetic tasks and computations. As a result, they may have had negative attitudes toward mathematics and fell further behind. Gagnon and Maccini stated that through problem solving and the use of manipulatives, students were able to make connections to the problem and internalize the mathematical meanings leading the students to understand the reasoning and discover the arithmetic associated with a solution (2001).

CHAPTER III

METHODOLOGY

In this study, the focus was on the students' changes in their approach to solving problems with rational numbers and the impact on their ability to correctly solve problems in contrast to a comparison group modeled in a quasi-experimental design (Shadish, Cook, & Campbell, 2002). The primary researcher was also the teacher of the experimental group. The intervention for the experimental group was the teaching of rational numbers using a structured problem solving strategy daily as a part of the curriculum. The control group did not receive the structured problem solving strategy but participated in a traditional teaching of rational numbers and problem solving. The two groups were then compared using null hypothesis testing, effect size estimates, and confidence intervals (CIs). The data were collected and instruments scored by trained third parties then cleaned data were provided to the researcher for analyses.

Participants

For this study, student participants were selected from two diverse school settings. In general, the participants were 6th grade students from public schools. Students were randomly selected from all the classes taught by the two teachers, one each in the experimental and control groups, although both groups were fully nested within teacher. Both genders were represented ($n_F = 70$; $n_M = 58$) and they were approximately 11 years and eight months old ($SD = 6$ months) for a total sample of 128. The sample represented the general composition of each school and district. The demographics and sample specifics are discussed in more detail in the sections on the Control and Experimental groups.

Control Group. The control group (n = 61) students were from an urban school district with similar demographics in Hispanics and the economically disadvantaged to the experimental group, to control for the fact that language could be a factor in the ability to problem solve. Of these participants, both genders were represented proportionately to the totals of students taught by each teacher (n_F = 33; n_M = 28) with 10 Caucasian students, 39 African American students, 10 Hispanic students and two Asian American students. The school was located approximately 100 miles from the experimental group's school district. The school district for the control group has a mobility rate of 30.8 percent (see Table 1 for a demographic comparison of control and experimental groups). Due to this mobility index it was likely, many students who started the study would have relocated before it was completed or moved into the district midway through the study, therefore, over sampling was used at the rate of 10% for a goal and final number for both groups to be no less than 50 each.

The control group students were randomly selected from all of the classes taught by the control teacher N = 123. All the students in the control teacher's classes were given all parts of the study initially. Then, each student was assigned a number one through six based on the roll of a fair die. From that point, the students were grouped by the number they were assigned and the die was rolled four more times to select which groups were to be included in the study.

The control group teacher administered the pretest and problem solving survey to her students prior to the instruction and review of the curriculum for ratios and proportions. Then after a three-week period, she administered the posttest and problem solving survey to the students.

Experimental Group. The experimental group (n =67) was located in a rural community in Texas with a diverse population and similar demographics in Hispanics and economically disadvantaged as the control group. Of these participants, both genders were represented (n_F = 37; n_M = 30) with 39 Caucasian students, 15 African American students, 12 Hispanic students and 1 Asian American.

I acted as the action researcher and the experimental group teacher. I administered the pretest and problem solving survey to my students before the instruction and review of the curriculum for ratios and proportions. After two weeks of daily problem solving activities based on ratios and proportions, I administered the posttest and problem solving survey. Each district allocates the teaching time for each content area. In this study the content is taught for a different period time between the two districts. Therefore, I did not choose to control this because the allocated time for teaching any content will differ from district to district, however, the same objectives must be covered during that allocated period of time. My function was similar to other action researchers (Fuys, 1984; Ross-Fisher, 2008; van Hiele-Geldof, 1957). More information about my research perspectives and interests are presented in the section about the teacher participants.

Table 1 *Demographic Representation for the Experimental and Control Groups*

Demographic	Experimental			Control		
	Sample	School	District	Sample	School	District
Mobility rating	3%	9.7%	13.8%	15%	30.8%	30.8%
Economic Disadvantaged	38.9%	45.8%	45.8%	48.5%	49.4%	49.4%
Caucasian	58.2%	56.4%	53.1%	16.4%	24.3%	36.6%
Hispanic	17.9%	19.4%	19.8%	16.4%	18.8%	20.1%
African American	22.4%	22.7%	25.5%	63.9%	51.6%	38.6%
Asian American	1.5%	1.6%	1.5%	3.3%	4.9%	4.0%

Note. School and district data retrieved from Texas Education Agency at http://www.tea.state.tx.us/cgi/sas/broker on August 23, 2008.

The Teacher Influence

This study takes place within two fully nested settings, that is, all the students were taught by one of two teachers. Therefore, it was important to discuss each of these teachers in detail to provide a context for the study and an understanding of impact they each might have imparted on the obtained effect estimates of the results.

Control group. The control group teacher had 27 years of classroom teaching experience and multiple degrees; a Bachelor of Music Education, a Master of Science in Music and a Doctorate of Philosophy in Curriculum and Instruction with emphasis in Mathematics Education. She spent 13 years teaching music at all levels of education, 24 years teaching 6th grade, 15 years teaching mathematics, and 2 years teaching language arts. The control group teacher used her traditional approach to problem solving and rational numbers without using the structured problem solving strategy administered to the experimental group. Her approach to problem solving included a continuous effort to incorporate problem solving into the curriculum throughout the school year. The problem solving method the control group was taught focused on a teacher made graphic organizer (See Appendix A) that helped students organize their solution while allowing them to select several problem-solving methods (S. Matteson, personal communication, August 23, 2008). The control group was taught rational numbers by activating prior knowledge of rational numbers as a building process. The teacher incorporated vocabulary skills by requiring students to define terms using textbooks and reference materials based on the lesson. The teacher also incorporated relationships to various forms of rational numbers, proportionality, and applications for rational numbers. She used modeling, manipulatives, and abstract representations of rational numbers for instruction and practice.

Experimental group. As the experimental group teacher, I had a Bachelor of Science in Interdisciplinary Studies with and emphasis in Mathematics and Science Education for Middle School and was pursuing an advanced degree similar to other action researchers (Ross-Fisher, 2008; van Hiele-Geldof, 1957). I taught 6^{th} grade mathematics for 2 years. I received training in problem solving strategies and activities as a part of my undergraduate coursework. During this experience, problem solving and problem posing activities were used with me to contextualize my understanding of the problem solving process. I worked with Polya’s (1957) problem solving process that included *understanding the problem*, *devising a plan*, *working the plan* and *reflecting*. I found that the reflection offered an opportunity to critique the solution and find additional methods for a solution but it did not ask for explanation or the reasoning for the solution. I speculated that an explanation of the steps to a solution would provide students a deeper more meaningful understanding of rational numbers.

After undergraduate school as part of my action research, I developed the problem solving strategy requiring students to explain their solutions. I earned the College of Education and Human Development's Outstanding Paper Award for my theoretical paper discussing the structured problem solving strategy. I worked through my problem solving strategy with problems based on rational numbers perfecting the problem solving strategy. Then, I scored my work, as well as, work from the pilot study using a variety of rubrics (Danielson, 1997; Van de Walle, 2007). After scoring the problems in multiple ways, I used a rubric developed by Danielson as a model to develop my rubric. This experience provided an opportunity for me to see varied levels of problem solving and scoring to determine which method was best for my study.

Instructional Strategy Development

The structured problem solving strategy is designed to assist students in moving through the steps of problem solving and provides a structured process for students to follow. The structured problem solving strategy (See Appendix B) is influenced by Polya's problem solving model (1957) and the ideas of Cifarelli and Cai (2005a, 2005b) that explanation and rationalization bring stronger meaning to the problem solving process. The combination of these researcher's ideas shaped the structured problem solving strategy for this study. It contains the four major steps of Polya's problem solving model; understand the problem, devise a plan, work the plan, and look back. In addition to Polya's problem solving model, the structured problem solving strategy required students to explain and rationalize their solution and process similar to Cifarelli and Cai.

Instructional Strategy

Between the pretest and posttest, the students in the experimental group received instruction in problem solving and were given ten daily problem-solving exercises. I introduced problem solving to the students initially by providing them with the structured problem solving strategy. I demonstrated solving a problem with the students using the method; explaining and focusing on the parts of the problem solving strategy. This introduction was written in notes by the students and administered as whole group instruction. Modeling was used throughout the two week period as well has having students model the strategy when they presented their solutions.

The students were guided through their first set of daily exercises (See Appendix C) the following day. The teacher provided modeling and support for the students during whole group instruction for the first three exercises. The teacher worked the problems by prompting the class through the steps of the problem solving strategy and modeling the expected explanations on the board. Class discussions were lead by the teacher to probe student understandings and misconception and to explore nuances of the problem. The teacher acted as a facilitator to provide support and help students progress through the problem solving steps

Once students were proficient, the teacher implemented a small group instructional strategy called *Think-Pair-Share* for the next four exercises (Kagan, 1994). The students were given two minutes to *think* about the problem and make notes about *understanding the problem*. The students were encouraged to read the problem several times and identify the information they know. The students were then paired with a partner for the *pair* in the small group activity. They were provided five minutes to make a plan and work through the problem solving strategy in the exercise with their partner. The teacher interacted with the pairs of students that were struggling to provide prompting as needed. When the timer rang after five minutes, the students were placed into groups of four for the *share* portion of the small group strategy. The students were given five minutes to *share* their solution and process within the group of four. While students were in groups, the teacher encouraged students to explore alternate solutions and methods as the groups discussed and wrote their looking back in the structured problem solving strategy. The teacher prompted students to talk about how they planned on solving the exercise and what approach they used to reach a solution, then leading to how the answer was realistic or correct. Following the group discussion, students were given four minutes to write their explanation of their solution detailing their reasoning for the steps in their plan and any changes they discovered during their look back writing in the small group setting. The teacher conducted a class discussion after the explanation writing. Students shared their solutions, as well as, their varied approaches to the problem. The implementation of the structured problem solving strategy from beginning to end each day was approximately fifteen minutes.

After using small group instruction for four exercises, the students worked the three remaining problem-solving activities independently; with the teacher leading class discussions after students worked through the exercises. Students were encouraged to share ideas and solution methods during the class discussion and were given five minutes to write their explanation about the discussion. At this point many students finished the exercises in approximately ten minutes although the majority of students still required the full fifteen-minute time frame.

The teacher led the experimental group through problem posing activities on the third, fifth and seventh day that the daily problem solving exercises were implemented,. With these problem-posing activities, students were told to create a problem or situation where ratio, proportion or percents would apply. Students were placed in pairs and encouraged to make these problems applicable to their own lives and experiences. After allowing five to seven minutes to create the problems, students exchanged problems with other groups and solved using the structured problem solving strategy within their pair. After the students solved each other's problems, the teacher led a class discussion where students discussed the variety of situations and experiences where ratios, proportion or percents were applicable. Students were encouraged to make inferences about where they may encounter the need for knowledge about these mathematical ideas in their future.

Survey, Materials, and Assessment

It was necessary to develop a survey, instructional materials and an assessment in order to evaluate the structured problem solving strategy. The survey estimated student attitudes toward problem solving and it was used to determine if attitudes were influenced by the structured problem solving strategy. The instructional materials were created to teach the structured problem solving strategy to the experimental group. The assessment was used to measure change in problem solving with rational numbers for both the experimental and control groups.

Survey. A survey about problem solving was designed (See Appendix D) to gather information about student experiences and attitude. Through reflection, I developed and modeled ten items on the survey after the Likert scale (i.e., Thompson, 2006). The survey was on a 5 point scale (1 = strongly disagree, 5 = strongly agree) assessing students attitude, applications and uses of problem solving. The researcher designed a survey that was administered to both control and experimental group before and after the pre/post test was administered. This was used to evaluate changes in student attitudes toward problem solving.

Materials. Materials were generated: The structured problem solving strategy and problem solving exercises were created to facilitate the teaching and learning of the approach. The traditional problem solving instructional items were based on state minimum skills test but more robust items were used for instruction. The test however, mirrored the items used on the state's high stakes test. Several problem-solving exercises were developed as a daily warm-up (See Appendix C). The problem solving exercises were designed to support the state curriculum focused on ratios and proportions. The exercises were completed at the beginning of class as a way to spark the students' interest. These exercises were developed and modeled after the practices of action research settings (Ross-Fisher, 2008; van Hiele-Geldof, 1957).

Assessment. In order to evaluate the students' progress, a test was designed to be used pre and post (See Appendix E) to measure the students' change in their problem solving ability with rational numbers. This pre/post test was developed by the researcher in alignment with action research practices that were explored in preparation for this study (Avison, Lau, Myers, & Nielsen, 1999; Ross-Fisher, 2008; van Hiele-Geldof, 1957). This test was carefully aligned to the teaching of ratios and proportions with six items aligned to the *Texas Essential Knowledge and Skills* (TEA, 2006) taught during this time frame. The items addressed problem solving with proportionality between similar geometric figures, converting fractions to percents, proportionality with a given ratio, expression of a ratio, and appropriate application of ratios.

A requisite scoring rubric was developed to score the problem solving exercises and the pre/post tests. The rubric (See Appendix F) allowed students to see how they were being scored and valued student responses based on mathematical accuracy, approach, and explanation for each problem. Students were scored on a scale of one to four in each category, four showing the most completion. In the problem solving strategy, the mathematical accuracy mapped to the *look back* section, the approach mapped to the *make a plan* and *work the plan* section, and the explanation mapped to the *explanation* section. The rubric did not account for the students' ability to understand the problem from the structured problem solving strategy. Students should have demonstrated their understanding of the problem in their explanation.

After implementing daily problem solving exercises for two weeks, ten class periods, the students were given a posttest (See Appendix E). This test was identical to the pre-test. The posttest was used for dual purposes of pre-test and post-test. Each item on the pretests and posttests were scored in all three categories of the rubric for each item on the test. Then a mean score for mathematical accuracy, approach and explanation were obtained for each test to use in the data analysis. The two tests were compared to evaluate change in their problem solving abilities.

Each item on the pre/posttest was scored by one of three raters; with the action researcher serving as the rater for the control group and two additional raters scoring the experimental group. The raters were trained on using the data from the students who were in the control group and not selected to participate in the study and the student data that was collected from the pilot study. Inter-rater reliability was established at 83% by all raters scoring pilot data and discussing where there were discrepancies and likenesses prior to the scoring of the pretest. The pretest for the experimental group was then divided alphabetically between the two experimental raters and all pretests were rated based on the terms created in this training. All the raters met a second time to establish inter-rater reliability at 83% prior to the scoring of the posttest. At this time the posttests for the experimental groups were divided alphabetically into two groups and the two experimental raters scored the opposite group from what they had scored for the pretest and all the raters scored the posttests based on the terms created in this training.

Procedure

Several steps were involved in the design of this study that began by defining what students thought of problem solving. The Likert-scale problem solving survey provided the researcher an opportunity to examine students' perspectives and previous encounters with problem solving. The survey was given to all students participating in the study prior to the implementation of the structured problem solving strategy as a tool to assess students' perceptions on attitude, uses and applications of problem solving.

Training. The teacher of the control group did not receive any training in the structured problem solving strategy and was not shown the activities. To determine the control teacher's perspective on problem solving and interview was conducted to assess her training and teaching methods of problem solving and rational numbers. The control teacher was trained in heuristic methods for problem solving through a variety of professional development opportunities. The graphic organizer that was developed by the control teacher was loosely based on Polya's problem solving model (1957). The control teacher had been trained on the four steps of Polya's process but did not directly implement the model and was not trained in other problem solving models.

The researcher and experimental teacher conducted a pilot study as part of the training for implementing this study. This allowed the researcher the opportunity to use the problem solving strategy and gain an understanding of the process and issues that needed to be addressed prior to the implementation of the study. The researcher received instruction on methods of modeling problem solving and teaching the structured problem solving strategy through professional development workshops and interactions with mentors. The pilot study provided training on scoring the pre/post test, as well as, examples of varied levels of performance with the problem solving exercises. The researcher was instructed in strategies for implementing problem solving independently, small group, and whole group instruction through undergraduate studies and through continuing education workshops.

As these procedures were implemented, the study began to generate measurable data for the researcher. With the assistance of the statistical software, SPSS, the researcher created a compilation of the data and began investigating the results of the study.

CHAPTER IV

RESULTS

The pretests and posttests were evaluated on three constructs, students' mathematical accuracy, approach to the problem, and explanation. Each student received a score for each construct on each of the assessments. This chapter presents the results from the data analyses. These results are organized by research question.

Each analysis uses α set to $p < .05$. Although statistically significant results indicates meaningful results, statistical significance does not show the magnitude of difference, the importance, or how much of the difference can be attributed to the experimental condition. Thus, effect sizes and CIs were used in alignment with the American Psychology Association (APA) recommendations for reporting research study results (APA, 2001; Thompson, 2007; Wilkinson & the APA Task Force on Statistical Inference, 1999).

Does the Implementation of a Structured Problem Solving Strategy Affect the Understanding of Rational Numbers for Middle Grades Participants?

Based on pretest to posttest comparison of the experimental group, the first research question addressed, does the implementation of a structured problem solving strategy affect the understanding of rational numbers for middle grades participants?

Experimental Group Results

The experimental group data were analyzed using a multivariate analysis of variance (MANOVA) comparing pretest means for mathematical accuracy, approach and explanation with the posttest means for the three constructs. Statistical significance between the mean scores for mathematical accuracy and approach were evident at p = .043 for mathematical accuracy and p = .01 for approach and no statistical significance for explanation was found at the level of $\alpha = .05$.

Using the formula developed by Cohen (Thompson, 2006), the effect size based on Cohen's d was performed on the means and standard deviations for mathematical accuracy, approach and explanation scores where $d = (M_E - M_C) / SD_{POOLED}$ are reported. Cohen's d is said to be a small effect size when d = .2, a medium effect size when d = .5, and large when d = .8 (Thompson, 2006).

The Cohen's d effect sizes for the experimental group were found for the scoring constructs with mathematical accuracy d = .55, a medium effect size, approach, d = .86, a large effect size, and explanation .44, a slightly smaller effect than medium. These results indicated that the implementation of the structured problem solving strategy not only increased students' ability to solve problems with rational numbers but that the structured problem solving strategy improved students problem solving skills. The strongest effect size was in *approach* that maps back to *understand the problem*, *devise a plan* and *work the plan* of the mathematics in the structured problem solving strategy.

To determine if the structured problem solving strategy differentially influenced various subgroups analyses were used to examine the effects. Subgroups were identified based on various demographics and explored for statistical significance.

Ethnicity. Viewing the 95% CIs in Figure 1, a statistically significant difference (p = .002) in the performance of mathematical accuracy of African American and White students can be observed. The 95% CI indicates that μ can be found within the bounds of this interval 95% of the time, where the bounds are estimates of the parameters and the margin of error is the distance from the mean to the bound (Cumming & Finch, 2005). When viewing the 95% CIs for ethnicity, there is a noticeable gap between the upper bound of African American students and the lower bound of the White students. The intervals are approximately the same height showing that these two groups within the sample have a similar variance. Therefore, the gap between the intervals may be explained by the African American students performed more computational errors leading to an incorrect solution in their mathematical accuracy scores.

Economically Disadvantaged and PreAP/Gifted and Talented. Based on the 95% CIs in Figure 2 and 3, there is a statistically significant difference ($p < .001$) in mathematical accuracy scores and in approach scores ($p = .005$) in the performance of students who are economically disadvantaged and students that are identified as PreAP/Gifted and Talented. While viewing the 95% CIs in Figure 2 and Figure, the interval indicates that μ can be found within the bounds of this interval 95% of the time, where the bounds are estimates of the parameters and the margin of error is the distance from the mean to the bound (Cumming & Finch, 2005).

When viewing the 95% CIs for Economic Disadvantage and PreAP/Gifted and Talented on Mathematical Accuracy Means on the Posttest for (Figure 2), there is a noticeable gap between the bounds of the Economically Disadvantaged, Non-PreAP/Gifted and Talented group and the PreAP/Gifted and Talented group, as well as, a gap between the Non-Economically Disadvantaged, Non-PreAP/Gifted and Talented group and the PreAP/Gifted and Talented group. The intervals are approximately the same height showing that these two groups within the sample have a similar variance. Therefore, the gap between the intervals may be explained by the PreAP/Gifted and Talented group was more accurate in their computation leading to more correct solutions.

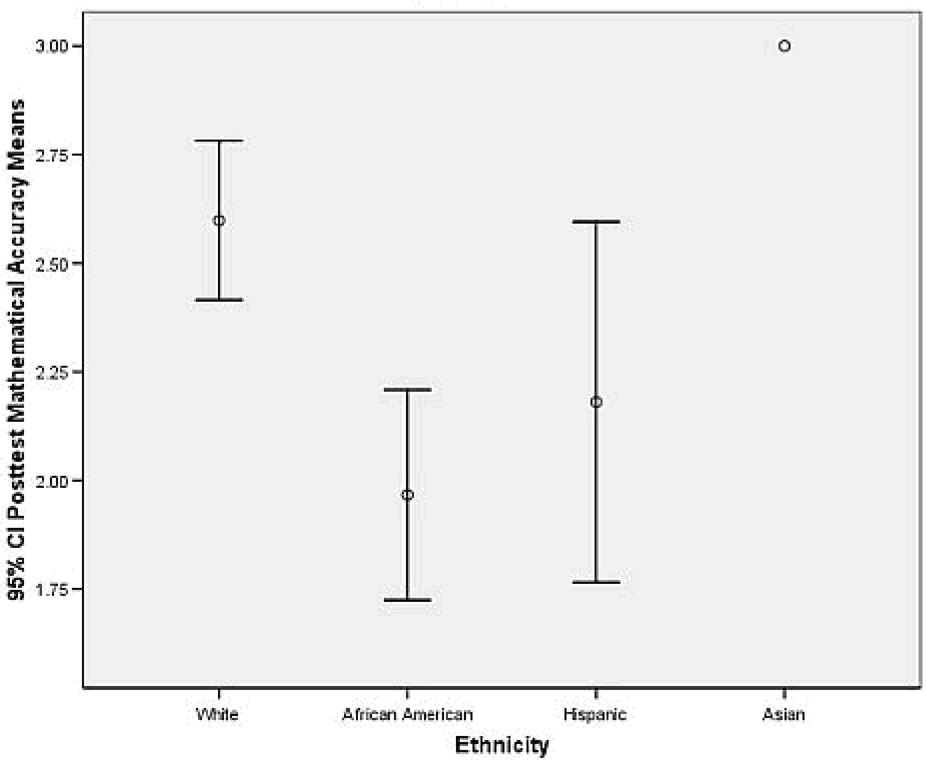

Figure 1 95% Confidence Interval for Ethnicity on Mathematical Accuracy Means on the Posttest

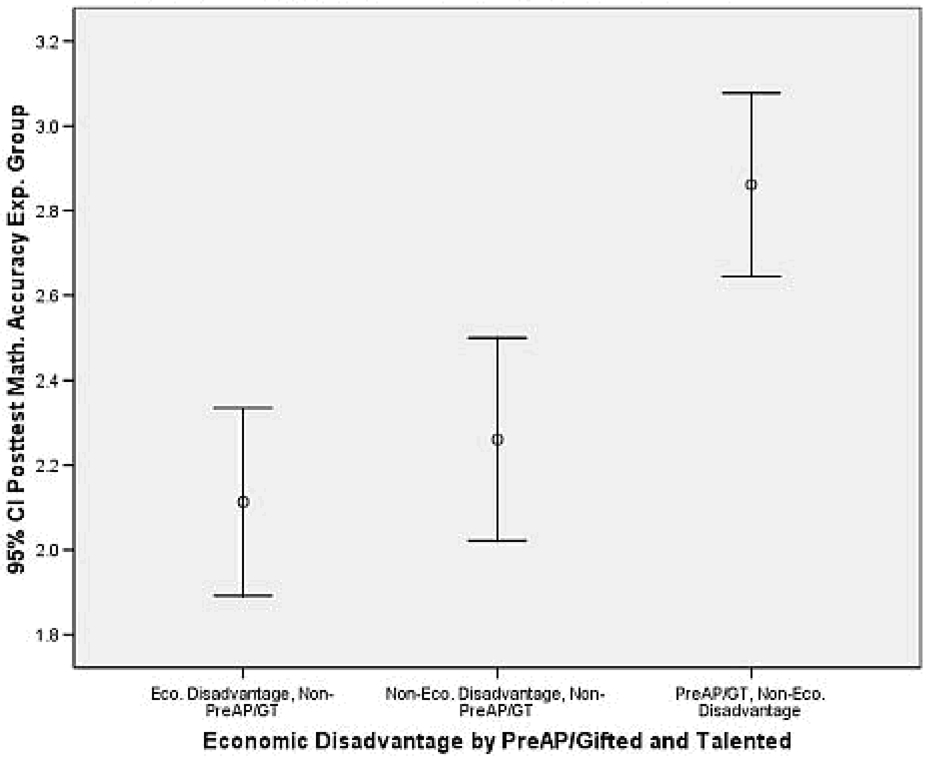

Figure 2 95% Confidence Interval for Economic Disadvantage by PreAP/Gifted and Talented on Mathematical Accuracy Means on the Posttest

In viewing the 95% CIs for Economic Disadvantage by PreAP/Gifted and Talented on Approach Means on the Posttest (Figure 3), a gap is present between the bounds of the Economic Disadvantaged, Non-PreAP/Gifted and Talented group and the PreAP/Gifted and Talented group. The Non-Economic Disadvantage, Non-PreAP/Gifted and Talented group overlap the upper bound and the lower bound of the PreAP/Gifted and Talented group. Although this overlap is slightly more than half the margin of error on the Non-Economic Disadvantage, Non-PreAP/Gifted and Talented group, it is less than half the margin of error on the PreAP/Gifted and Talented group.

As a result of the margin of error being larger on the PreAP/Gifted and Talented group it is not safe to claim a statistically significant difference for these two groups on approach. However, the Economic Disadvantaged, Non-PreAP/Gifted and Talented group and the PreAP/Gifted and Talented group does support the statistically significant (p = .005) findings in the MANOVA. The margin of error for the PreAP/Gifted and Talented group is larger indicating a larger variance in the sample that may be a result of students skipping steps in their solution approach or adding unnecessary information to their solution approach.

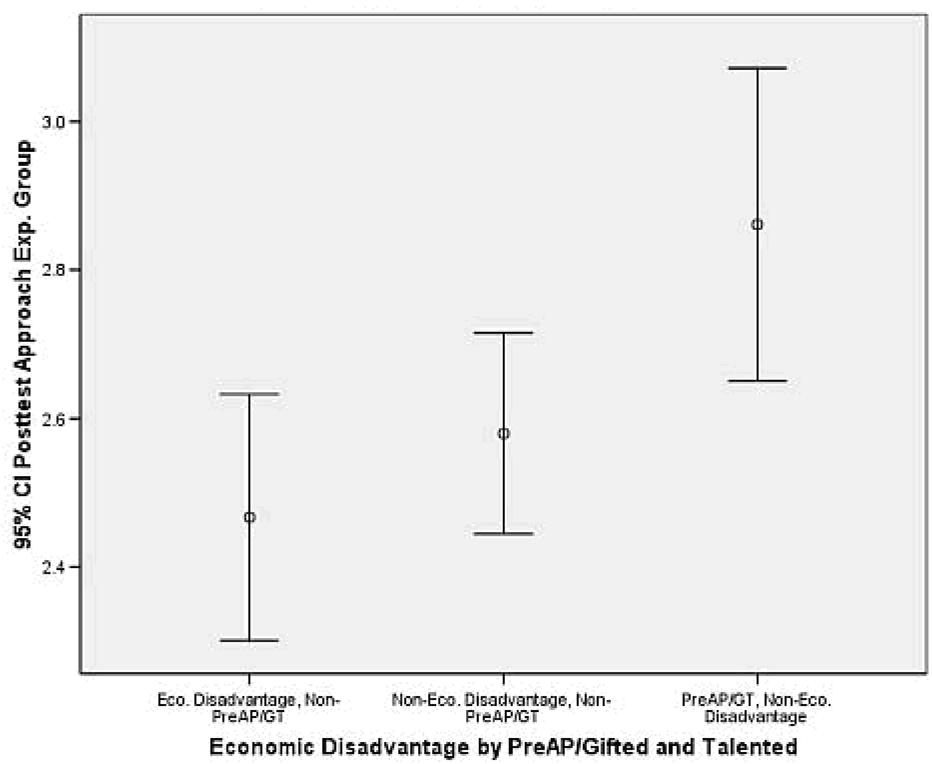

Figure 3 95% Confidence Interval for Economic Disadvantage by PreAP/Gifted and Talented on Approach Means on the Posttest

Does the Implementation of a Structured Problem Solving Strategy Affect Students' Performance in Problem Solving?

In response to the second research question, "Does the implementation of a structured problem solving strategy affect students' performance in problem solving?" the scores for mathematical accuracy, approach and explanation were all evaluated between the pretest to the posttest in the control group, in comparison of the control group to the experimental group on the posttest, and question 6 of the pre/posttest.

Control Group Results

Looking at the control group the group as a whole showed that there was some improvement over the course of the study. By viewing the 95% CIs in Figures 4-6, the control group improved in each area of the rubric except for explanation. However, based on an analysis of the means in a MANOVA, there were no statistically significant results with the control group when comparing each score from the rubric's pretest to posttest. This can be explained in the 95% CIs. By viewing the 95% CIs for Pretest to Posttest on Mathematical Accuracy (Figure 4), the bounds for the prettest and posttest for the control group overlap beyond half of the margin of error supporting $p > .05$. However, the 95% CI does show that the control group improved from pretest to posttest and that μ for the group will fall within the bounds of the 95% CI of the time.

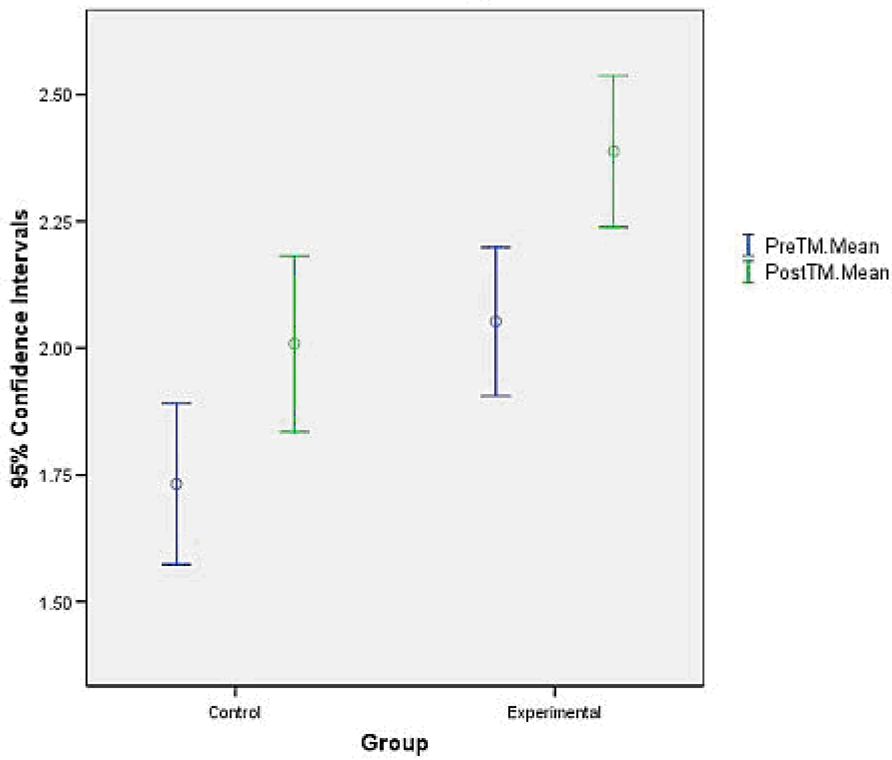

Figure 4   95% Confidence Interval for the Pretest to Postest on Mathematical Accuracy Means

Based on the 95% CIs for Pretest to Posttest on Approach Means (Figure 5), where the bounds represent the parameters, μ is located within the bounds 95% of the time and the margin of error is the distance from the mean to the bound, the CIs for the control group support that a statistically significant difference ($p > .05$) is not present on the approach construct of scoring. Despite that there is not any statistical significance; the 95% CI does illustrate the control groups' improvement on this scoring construct.

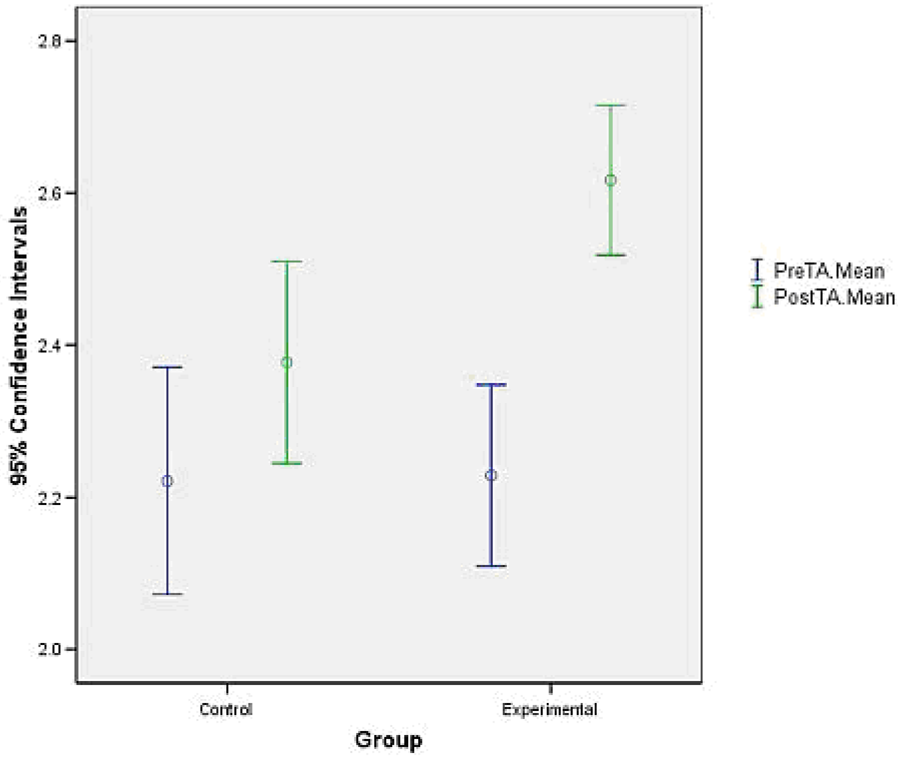

Figure 5 95% Confidence Interval for the Pretest to Posttest on Approach Means

Viewing the 95% CI for Pretest to Posttest on Explanation Means (Figure 6), the bounds for the pretest and posttest for the control group overlap almost the length of the margin of error supporting the findings that there was not a statistically significant difference ($p > .05$) in the performance of the control group. Although there was not a statistically significant difference in the pretest and posttest, the 95% CIs do support the findings that the control group slightly decreased in performance.

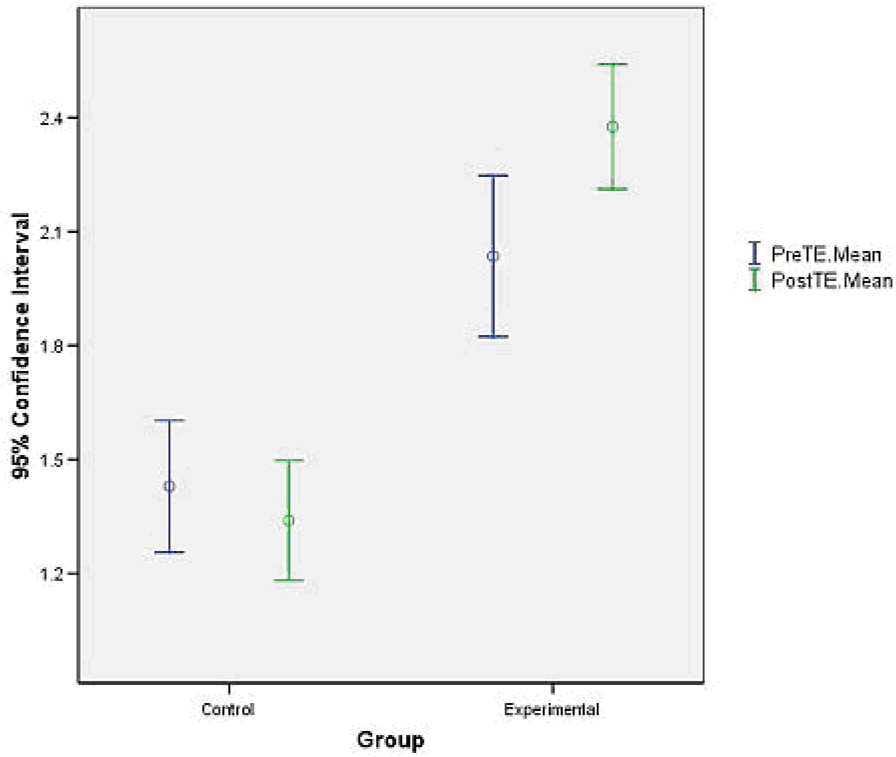

Figure 6   95% Confidence Interval for Pretest to Posttest on Explanation Means

The effect sizes reported in Table 2 based on Cohen's *d*. The control group has an effect size for the explanation score that is negative indicating that the control group was slightly less in their explanation from the pretest to the posttest. Although the explanation showed a negative effect size, the mathematical accuracy and the approach have a positive effect size indicating some improvement over the course of the study.

Table 2 *Cohen's d for the Pretest to Posttest*

Scoring Categories	Control Group	Experimental Group
Mathematical Accuracy	.43	.55
Approach	.28	.86
Explanation	-.14	.44

The means and standard deviations for both groups are reported in Table 3.

Control Group to Experimental Group Results

The scores for the posttest were analyzed in a MANOVA with mathematical accuracy, approach and explanation as dependant variables and independent variable was the group and the pretest means of mathematical accuracy, approach and explanation were used as covariates. This analysis produced a significantly different difference for mathematical accuracy, $p < .001$, approach, $p = .04$, and explanation, $p < .001$. While comparing the posttest for the control and experimental groups in all the levels of the grading rubric, three effect sizes were produced and reported in Table 4.

Table 3 *Mean and Standard Deviations of Control and Experimental Groups for Pretest and Posttest*

	Mean		Standard deviation		Sample size	
Variable	Control Group	Exp. Group	Control Group	Exp. Group	Control Group	Exp. Group
Pretest						
Mathematical accuracy	1.73	2.05	.62	.60	61	67
Approach	2.22	2.28	.58	.49	61	67
Explanation	1.43	2.03	.68	.87	61	67
Posttest						
Mathematical accuracy	2.01	2.39	.68	.61	61	67
Approach	2.38	2.62	.52	.45	61	67
Explanation	1.34	2.38	.62	.67	61	67

Cohen's *d* is said to be a small effect size when *d* = .2, a medium effect size when *d* = .5, and large when *d* = .8 (Thompson, 2006). Based on this statement, the effect size for mathematical accuracy and approach are medium and the effect size for explanation is very large. The experimental treatment can account for to .35 to 1.0 point increase in the varied areas of scoring by multiplying the standard deviation by the effect size of each of the scoring categories. Mathematical accuracy accounts for .36 points increase, approach accounts for .35 points increase and explanation accounts for 1.08 points increase. While keeping in mind that the scores were from one to four, this is responsible for up to a 25 percent increase in the area of explanation. Therefore, the experimental implementation of a structured problem solving strategy shows that it positively impacted participant understanding of rational numbers.

Table 4 *Cohen's d for Posttest Comparing Experimental Group to Control Group*

Score Categories	Cohen's *d*
Mathematical Accuracy	.59
Approach	.52
Explanation	1.60

CIs were calculated and graphed, as well, showing the 95% CIs (See Figures 4 – 6). When viewing the 95% CIs for Pretest and Posttest on Mathematical Accuracy

Means (Figure 4), there is a small gap between the bounds of the control group posttest and the experimental group posttest indicating the statistically significant difference found in the MANOVA of $p <.001$, where the bounds indicate the parameters where μ can be found 95% of the time. Based on the 95% CIs for Pretest to Posttest on Approach Means (Figure 5), the bounds for the posttest for the control group overlaps the bounds for the posttest for the experimental group slightly indicating a statistically significant difference ($p = .04$), where μ is located within the bounds of the CI 95% of the time. The most statistically significant difference ($p < .001$) is present in Figure 6, 95% CI for Pretest to Posttest on Explanation Means between the posttest of the control group and the posttest of the experimental group. The gap between the bounds of the intervals is more than the margin of error apart supporting the findings of the effect size and the MANOVA.

Problem Solving Question in Pre/Posttest

This question on the pre/posttest was designed as an example for when not to use rational numbers in the solution method. Question 6, specifically, was used test for a statistically significant difference between the pretest and posttest in the experimental group on all three categories of the grading rubric, mathematical accuracy, approach and explanation. The scores were analyzed in an ANOVA showing statistically significant differences in the area of approach, $p < .043$. There was not a statistically significant difference in the scores for mathematical accuracy and explanation. The means and standard deviations for all score categories were used to calculate the effect size based on Cohen's d producing an effect of $d = .18$ for mathematical accuracy, $d = .33$ for approach, and *d* = .54 for explanation. Although these effect sizes are small, the effect size provides evidence that the structured problem solving strategy positively improved the students' problem solving skills.

Can the Implementation of a Structured Problem Solving Strategy Influence Students' Attitudes About Problem Solving?

In response to the third research question, "Can the implementation of a structured problem solving strategy influence students' attitudes about problem solving?" A Likert survey was administered to the participants in the study. The statements from the Likert survey were grouped into factors from an exploratory factor analysis (EFA) using the statistical packages for the social sciences SPSS and guidelines for reporting EFA (Henson, Capraro, & Capraro, 2004). The EFA was performed using principal component analysis with varimax rotation. Three factors were extracted using the eigenvalue-greater-than-one rule and a scree plot test (see Figure 6). Table 5 contained the eigenvalues for the three factors, as well as, the first component not retained.

Table 5 *Factor Analysis of the Problem Solving Survey*

	Initial Eigenvalues		
Component	Total	% of Variance	Cumulative %
1	3.20	32.02	32.02
2	1.41	14.11	46.14
3	1.12	11.24	57.37
4	.89	8.89	66.27

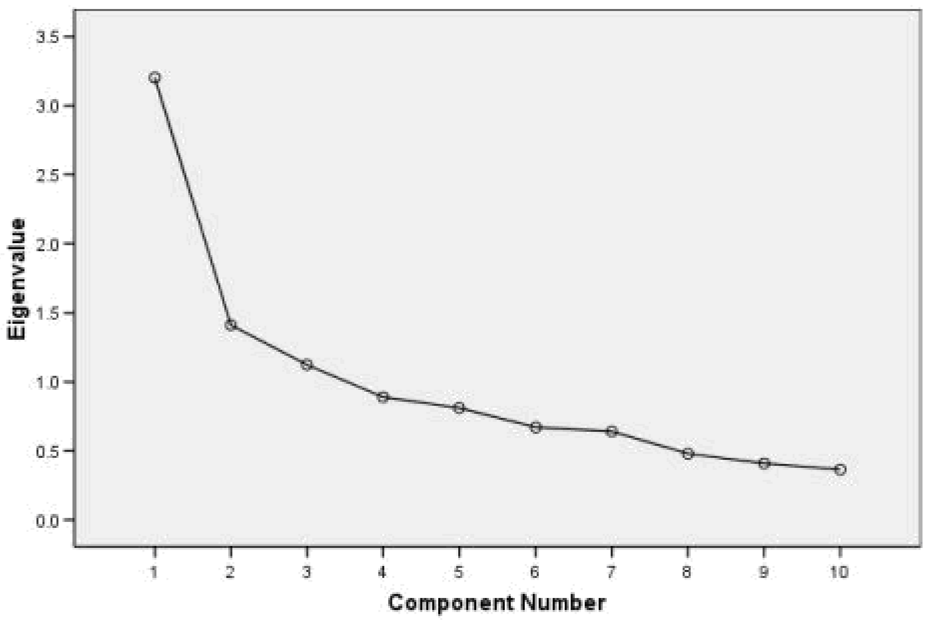

Figure 7 Scree Plot

The scree plot supports the eigenvalue-less-than-one rule with three factors. The pattern matrix, Table 6, show the pattern/structure coefficients for each of the components and to which component each question was aligned.

Table 6 *Pattern/Structure Coefficients*

	Component		
Statements	I	II	III
2	.771	.198	.005
6	.679	.286	.190
3	.664	-.051	.138
4	.597	.165	.065
5	.156	.831	.066
10	.140	.750	.017
1	.141	.746	.125
9	.058	.007	.821
7	.086	.088	.793
8	.402	.171	.517

Note. Extraction Method: Principal Component Analysis. Rotation Method: Varimax with Kaiser Normalization.

Using the pattern/structure coefficients on the ten Statements in the Likert posttest with statement ten being reverse scored, the statements were grouped into one of three factors that were named, uses (Component I), attitude (Component II) and applications (Component III) of problem solving. Statement 8 clearly has a strong relationship to attitude and applications, based on the nature of the statement, the researcher chose to place the statement in both factors for uses and applications. The remaining statements were grouped with the factor that showed the greatest value of a relationship.

The statements were grouped into three factors based on the results and named *uses*, *attitude* and *applications of problem solving*. The scores for each statement in each factor were then combined to find a mean value for each factor in the pretest and posttest per student. These means were analyzed in MANOVA. The factor for uses of problem solving found a statistically significant relationship between the pretest and posttest with $p < .032$ and Cohen's d value of .28. There was a statistically significant relationship for the attitude factor of problem solving between the pretest and posttest with $p < .012$ and Cohen's d value of .15. The factor for applications of problem solving did not result in a statistically significant relationship between the pretest and posttest with $p < .05$.

CHAPTER V

SUMMARY AND CONCLUSIONS

In this chapter the results are discussed, as well as, a summary, and conclusions addressing each of the research questions. After conclusions, this chapter goes on to explore implications for future research in the area.

Summary

This study was designed as an action-research study (Ross-Fisher, 2008; van Hiele-Geldof, 1957) in a quasi-experimental model (Shadish et al., 2002) to examine the idea that problem solving can provide a deeper meaning and understanding of rational numbers. Aligned with NCTM (2000), the study indicates that problem solving can improve the mathematical accuracy, approach and the explanation of rational numbers that are focused on rates, ratio, proportion, and percents. In a comparison of a control group and an experimental group that received instruction with a structured problem solving strategy, the study showed a statistically significant difference in the performance of these two groups. When examining the groups and data more closely, the study shows that the structured problem solving strategy not only improved students ability to understand and use rational numbers but the structured problem solving strategy also improved students' problem solving skills and their attitude towards problem solving.

Problem Solving and Rational Numbers

Research has shown that students struggle with rational numbers and proportional reasoning (Brown & Quinn, 2006; Cramer & Post, 1993b; Cramer, Post, &

delMas, 2002; Kilpatrick, Swafford, & Findell, 2001; Lamon, 1993; Lesh et al., 1987). Based on this evidence, this study combined proportional reasoning and rational numbers with problem solving in an effort to combine necessary understanding of rational numbers with an aspect that relates to students' previously constructed knowledge through problem solving (Bottge et al., 2006; Jurdak, 2006; Stillman & Galbraith, 1998; Stoyanova, 2003). By relying on the structured problem solving strategy, students were able to explore rational numbers and manipulate numbers into various representations. Traditionally rational numbers are only shown with algorithmic sequences; this might prevent students from relating to the numbers or from being able to apply them to individual experiences (Lappan, 2006; Lesh et al., 1987).

When students began using the structured problem solving strategy, they showed hesitation and they were uncomfortable with what to do or how to apply the strategy similarly to Lubienski's (2000) findings. The Likert survey results showed findings supporting improvement in students' attitudes and confidence in their ability to problem solve. As students became more confident in their solutions they also became more confident in their ability to solve the problems.

After the first three exercises as a whole group, students began to realize that there was no wrong way to approach the problem as long as they could justify and explain their answer (Cifarelli & Cai, 2005a, 2005b). Quickly, students began to move through the first step of *understand the problem* of the structured problem solving strategy and begin developing individualized plans for the *make a plan*. The students began to share and analyze plans with their peers in the Think-Pair-Share activity with the exercises, showing the most improvement in their explanation at this stage of the study (cf. Adibnia & Putt, 1998). When discussing which plan was the best method to solve the problem, students started to justify individual plans and rationalize why a procedure should be done. These discussions provided opportunity for growth within the experimental group. Students began to look at problems from various viewpoints. Simultaneously, some of the students began to internally *understand the problem*. As the students became more familiar with the structured problem solving strategy and gained deeper understanding of rational numbers, some students were writing their *understand the problem* as part of the plan for the solution while some students were internalizing the *make a plan* portion of the structured problem solving strategy and moving from *understand the problem* to *work the plan*. This is consistent with Kulm and Bussman (1980) where steps or phases can be skipped or integrated into other steps.

Students were encouraged by their solutions being evaluated as a process and being allowed to approach the problem using a variety of solution methods. The mathematical accuracy with their solutions began to improve during the later part of the study when small group Think-Pair-Share was employed, as well as, independent implementation of the structured problem solving strategy.

When students moved to working with the structured problem solving strategy independently, they were eager to begin the class discussion to discover if their solution and plan was used by other students. By anticipating another route to the solution and expanding on each other’s solution, students illustrated understanding and success with rational numbers and problem solving. Students began to self-correct their plans, solutions, and explanations before and during the class discussion in line with stage four in Piaget's work (1964).

Based on exploration of the data for the scores of explanation and approach for the structured problem solving strategy, student improvements in problem solving were evident. Initially, during the exercises, students struggled to create a plan and to explain and rationalize their solution. Many students had never been asked, "Why?" The explanation portion of the structured problem solving strategy forced students to evaluate beyond the algorithm and rationalize and justify their solutions. As students worked in small groups, their ability to justify and explain their approach improved showing understanding of the reasoning for performing the processes of the algorithm.

One of the problems on the pre/posttest was designed to show when not to use rational numbers and to highlight students' problem solving skills. Students showed multiple solution plans and processes that were not related to rational numbers. The data supports these assumptions by showing positive effect sizes and an increase in the means from pretest to posttest. Although the study focused on rational number concepts, the structured problem solving strategy provided an opportunity for students to learn a strategy to solve problems outside rational numbers and proportional reasoning.

PreAP/Gifted and Talented. Initially, the PreAP/Gifted and Talented students in the experimental group were the only students to have experience with problem solving in any systematic form. The experience with problem solving provided this group with an advantage and the least amount of improvement. In comparison with Lubienski's study (2000), the students in the PreAP/Gifted and Talented subgroup were not economically disadvantaged (see Figure 5) and carried similar openness and success with learning through problem solving. Although, Lubienski's high level students yielded the most success with problem solving, the students in this study did not mirror her results.

Middle School Mathematics Educators

In the modern world where high-stakes testing holds every educator accountable for their students' mastery of national and state mandated curriculum, it is necessary for educators to evaluate their students' retention of the content, as well as, students' ability to correctly reach a solution on the test. Educators are charged with the responsibility of preparing students conceptually for the next level of mathematics while providing an opportunity for the students to make connections to realistic situations. The mathematics educator is preparing students for higher level mathematics and future employment opportunities.

With this in mind, mathematical accuracy is of utmost importance. Students who go on to become physicians must be able to administer the correct dosage of medicine to their patients. Future structural engineers must be able to keep our buildings and homes structurally sound and safe. The lack of mathematical accuracy not only results in failure in a high-stakes testing environment but can lead to deadly outcomes in the work environment.

Student approach and explanation to a problem can influence the student's understanding of future mathematics. As educators teach ratios, rates, proportions, and percents, they must recognize that rational numbers appear in multiple representations in the environment outside of the mathematics classroom. As the skills with rational numbers are carried away, students will encounter a need for multiple approaches to be successful in future classes as well as the workplace. With the ability to rationalize and justify their answers, students learn to use critical thinking skills that will assist them with additional aspects of mathematics that will carry them on to solve additional problems in a realistic setting in the future (e.g., Cifarelli & Cai, 2005a).

Future Research

This study showed that the structured problem solving strategy can assist students to learn and understand rational numbers with more success and improve students' attitude toward problem solving. The next question is "Is the structured problem solving strategy transferable to other areas of mathematics?" Perhaps, future research can be conducted to pursue using the structured problem solving strategy to reach students struggling in the Measurement or Algebra strands. While rational numbers are a part of both of these strands, the questions have to be asked, "Will the problem solving knowledge of rational numbers transfer to these strands after the implementation of the structured problem solving strategy?" and "Will the structured problem solving strategy assist students in retaining the knowledge and understanding of rational numbers they gained during the study?"

Although retention of the concept is something that every teacher struggles with, when students have a deeper understanding and meaning of the concept the construction of the knowledge becomes part of the experience. Students should be able to tap into this web of knowledge by remembering the experience and using it in future situations.

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APPENDIX A

CONTROL GROUP PROBLEM SOLVING GRAPHIC ORGANIZER

<table>
<tr><td colspan="2">Known</td><td colspan="2">Unknown</td><td>The Strategies

Draw a Picture/Diagram
Look for a Pattern
Guess and Check
Act it out
Make a Table/List
Work Backwards
Write a Math Problem</td></tr>
<tr><td colspan="3">Create a Representation:
Picture/Diagram/Table/Rule/Graph</td><td colspan="2">Work (Use back if more room is needed)</td></tr>
<tr><td colspan="3">Final Answer (complete sentence)</td><td colspan="2">Check answer. Is answer reasonable?</td></tr>
</table>

APPENDIX B

STRUCTURED PROBLEM SOLVING STRATEGY

<table>
<tr>
<td>

<u>UNDERSTAND THE PROBLEM</u>

- Read the problem through twice.
- Identify the important information in the problem.
- **What are you looking for?**
- Can you draw a diagram/illustration of your problem?
- What are the key terms in this problem?
- Do you know all the definitions of the terms in the problem?
- Have you worked a similar problem?
- Do you have any previous solutions or experiences that may apply to the problem?

</td>
<td>

<u>DEVISE A PLAN</u>

- Create a plan for your problem.
- Write the plan out
- Are there any formulas that apply to this problem?
- How can the illustration help you?
- Will another related problem help with the solution?
- Can you use a previous plan?

</td>
</tr>
<tr>
<td>

<u>WORK THE PLAN</u>

- Apply the plan you created.
- Find a possible solution to the problem.
- Is there another solution to your problem?
- Have you used all of your data?

</td>
<td>

<u>LOOK BACK</u>

- Is there another way to solve your problem?
- Did you follow your plan?
- Is this problem applicable to another problem?
- Does your plan need to be modified or changed?

</td>
</tr>
<tr>
<td colspan="2">

<u>EXPLANATION</u>

- Explain your problem and solution. (about 3-5 sentences)
- Check your work. Does your answer check out?
- If your answer is incorrect, why?
- Is your answer realistic?

</td>
</tr>
</table>

Adapted from:

Polya, G. (1957). *How to solve it: A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

APPENDIX C

DAILY EXERCISES

Daily Exercise 1

An IV-bag with 1000ml of fluid is delivering fluid to a patient at the rate of 3 ml per minute. After the first three hours, how many milliliters of fluid remain in the IV-bag?

Daily Exercise 2

Eighteen acres of land sold for $27, 766.80. At that same rate, what is the cost of six acres of land?

Adapted from:
Herr, T., & Johnson, K. (1994). *Problem solving strategies: Crossing the river with dogs.* Berkley, CA: Key Curriculum Press.

Daily Exercise 3

Jenna jogs at a rate of six miles per hour. What is her rate in feet per second? Express your answer as a decimal to the nearest tenth.

Daily Exercise 7

Three out of seven students in Mr. Sullivan's first period class are boys. Mr. Sullivan has 23 students in his second period class, 26 students in his third period class, 24 students in his fifth period class, 22 students in his sixth period class and 25 students in his seventh period class. If this ratio applies to all of Mr. Sullivan's classes, how many boys does Mr. Sullivan teach?

Adapted from:
Chandler, K. L. (2004). 2004 – 2005 *Mathcounts school handbook.* Alexandria, VA: MathCounts.

Daily Activity 4

The table shows the results of a survey in different classrooms before a class election. In which classrooms, did you receive the same ratio of votes to the total number of votes?

Sixth Grade Student Body President Election

Classroom	Mrs. Martin	Mrs. Sewall	Mrs. Cline
Your Votes	13	10	12
Total Votes	26	20	22

Daily Activity 5

A photo 5 inches wide and 7 inches long is enlarged. The sides of the new photo are proportional to the original. The new photo is 14 inches wide. What are the dimensions of the new photo?

Daily Activity 6

An airplane flies 2,750 miles in 5 hours. Find the unit rate in miles per second. Round your answer to the nearest hundredth.

Daily Activity 8

A jar contains 20 white marbles, 30 black marbles and some red marbles. Half of the marbles are black. Find the ratio of white marbles to red marbles.

Daily Activity 9

A glacier moves 12 inches every 36 hours. About how far does the glacier move in one week?

Daily Activity 10

Suppose you are making a castle for miniature figures. A 6-foot-tall knight is represented by a figure that is 30 mm tall. Actual castle walls are about 30 feet high. How high should you make the walls of the model?

Adapted from:

Charles, R. I., Illingworth, M., McNemar, B., Mills, D., Ramirez, A., & Reeves, A. (2008). *Texas mathematics: Course one.* Boston: Prentice Hall.

APPENDIX D

PROBLEM SOLVING SURVEY

Circle the number that best describes your feelings for the following statements.

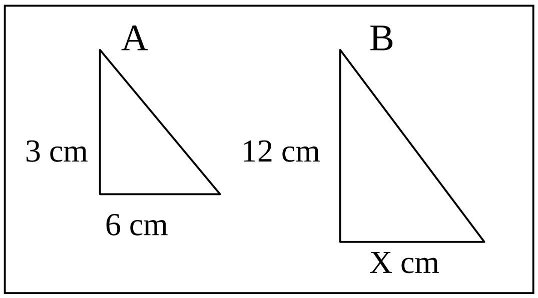
	☹		😐		☺
1. I like problem solving.	1	2	3	4	5
2. You can use problem solving outside of math.	1	2	3	4	5
3. You can use ratios and proportion in problem solving.	1	2	3	4	5
4. You can use problem solving in sports.	1	2	3	4	5
5. I am a good problem solver.	1	2	3	4	5
6. I have used problem solving.	1	2	3	4	5
7. Problem solving is required in most jobs.	1	2	3	4	5
8. I use problem solving every day.	1	2	3	4	5
9. I will solve problems at my job in the future.	1	2	3	4	5
10. I am a poor problem solver.	1	2	3	4	5

APPENDIX E

PRE/POSTTEST

SHOW ALL YOUR WORK AND EXPLAIN HOW YOU DECIDED WHAT TO DO!!!

1. Triangle A and triangle B are proportional.



   What is the ratio of the dimensions of triangle A to the dimensions of triangle B? What is the length of the missing base in triangle B? (TEKS obj. 6.3a)

2. Today there were 4622 fans at the soccer game. Last week, there were only 3095 fans at the soccer game. What is the percent increase in the number of fans from last week's game to today's game? Express your answer to the nearest whole number. (TEKS obj. 6.3b)

3. The ratio of an object's weight on Earth to its weight on the Moon is 6:1. The first person to walk on the Moon was Neil Armstrong. He weighed 165 pounds on Earth. How much did he weigh on the Moon? (TEKS obj. 6.3c)

4. The carvings at Mount Rushmore National Memorial in South Dakota are 60 feet from chin to forehead. The distance from chin to forehead is typically 9 inches long. The distance between the pupils of the eyes is 2.5 inches long. What is the approximate distance between the pupils in the carving of George Washington's head? (TEKS obj. 6.3c)

5. Write a ratio for the number of vowels to the number of consonants in the English alphabet. (TEKS obj. 6.3a)

6. Sara visits a farm and sees chickens and pigs in the barn. Sara noticed that there were 35 heads in the barn. She counted 100 legs. How many chickens and how many pigs were in the barn? (TEKS obj. 6.3c)

APPENDIX F

PROBLEM SOLVING SCORING RUBRIC

	Level One	Level Two	Level Three	Level Four
Explanation	Little or no explanation, or impossible to follow.	Explanation attempted, but difficult to understand.	Explanation fairly clear, but thinking process not always easy to follow.	Explanation very clear, and thinking process easy to follow.
Approach	Random and disorganized no systematic approach.	Some system apparent in the approach, however, it is difficult to follow.	Systematic and organized approach, but not well presented.	Highly systematic and organized approach; neatly and clearly presented.
Mathematical Accuracy	Many computational errors, leading to wildly erroneous conclusions.	Some computational errors, but allowing largely for accurate conclusions.	Virtually no mathematical errors; with accurate conclusions.	Completely accurate computation, allowing for accurate conclusions.

Adapted from:

Danielson, C. (1997). *A collection of performance tasks and rubrics: Upper elementary school mathematics.* Larchmont, NY: Eye on Education.

VITA

Krystal Baker Meredith
4116 Fawn Drive
Killeen, TX 76542

Education

Master of Science, Curriculum and Instruction, Texas A&M University, May 2009.

Bachelor of Science, Interdisciplinary Studies, Texas A&M University, May 2006.

Professional Certification

Mathematics and science grades 4-8 issued by the State of Texas.

Experiences

August 2008 – present	Killeen Independent School District	Killeen, TX
Palo Alto Middle School		
Grade 6 – Math Teacher		
August 2006 – May 2008	Brenham Independent School District	Brenham, TX
Brenham Middle School		
Grade 6 – Math Teacher		

Honors and Awards

Kappa Delta Pi honor society
National Society of Collegiate Scholars honor society
National Council of Teachers of Mathematics
Dean's Award, Educational Research Exchange, January 26, 2007

Presentations

Meredith, K. B. (2008, June). *Organizational habits of highly effective teachers*. Presented at the annual meeting of the Beginning Teacher Academy, Brenham, Texas.

Meredith, K. B. (2008, February). *The effects of implementing a problem solving strategy in a middle school classroom*. Presented at the annual meeting of the Southwest Educational Research Association, New Orleans, Louisiana.

Meredith, K. B. (2007, January). *The effects of implementing a problem solving strategy in a middle school classroom.* Presented at annual meeting of the Educational Research Exchange, College Station, Texas.